Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/3C**

Further Mathematics

Advanced

PAPER 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Year 1 Elastic Collisions in 1D - Impulse-momentum, PCLM

1. A particle A of mass $3m$ and a particle B of mass m are moving along the same straight line on a smooth horizontal surface. The particles are moving in opposite directions towards each other when they collide directly.

Immediately before the collision, the speed of A is ku and the speed of B is u.
Immediately after the collision, the speed of A is v and the speed of B is $2v$.

The magnitude of the impulse received by B in the collision is $\frac{3}{2}mu$.

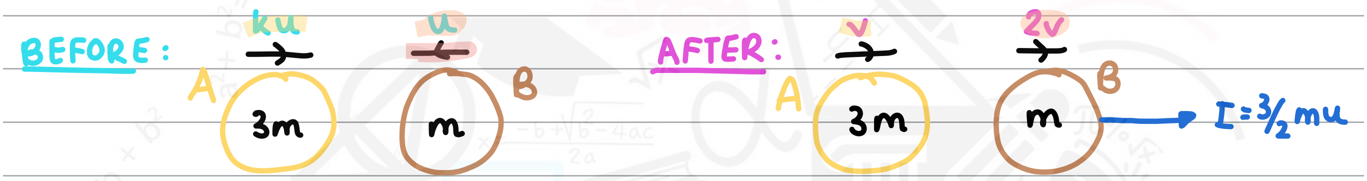
(a) Find v in terms of u only.

(3)

(b) Find the two possible values of k.

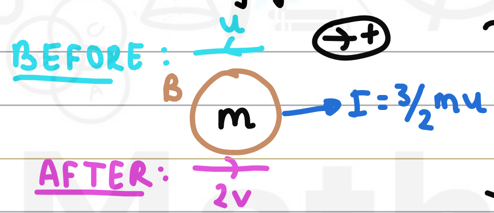
(5)

illustrating this elastic collisions in 1D diagrammatically - label the respective speeds, direction of motion, and the impulse



(a) could think to use PCLM and NEL as usual with elastic collisions in 1D - but too many unknowns - let's use the information given for B instead

... looking just at B:



we can use the Impulse-momentum formula to try give 'v' in terms of 'u'

formula: $I = m(\underline{v} - \underline{u})$

$\frac{3}{2}mu = (m)(2v - (-u))$

expand and cancel m's

$\frac{3}{2}\cancel{m}u = \cancel{m}(2v + u)$

$\Rightarrow \frac{3}{2}u = 2v + u$

want 'v', so collecting 'u's

$2v = \frac{1}{2}u$

$\div 2 \qquad \div 2$

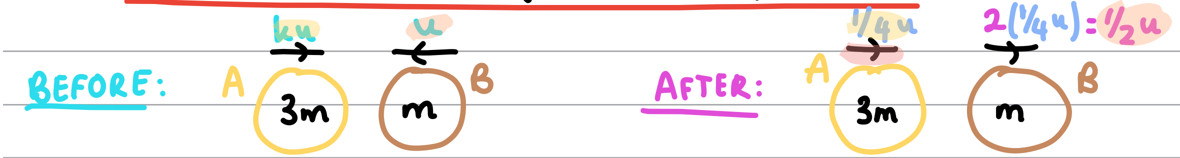
$\boxed{v = \frac{1}{4}u}$

(b) Now let's use PCLM - first, let's sub in $v = \frac{1}{4}u$ into the 'AFTER' diagram so

Question 1 continued

that we have one less variable to worry about

CASE 1: A doesn't change direction after collision



... subbing above into PCLM - i.e total momentum before the collision equals the total momentum after:

formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

sub into above

$$3\cancel{m}(ku) + \cancel{m}(-u) = 3\cancel{m}\left(\tfrac{1}{4}u\right) + \cancel{m}\left(\tfrac{1}{2}u\right)$$

cancel the m's and expand above:

$$3ku - u = \tfrac{3}{4}u + \tfrac{1}{2}u$$

collect like terms

$$3k\cancel{u} = \tfrac{9}{4}\cancel{u}$$

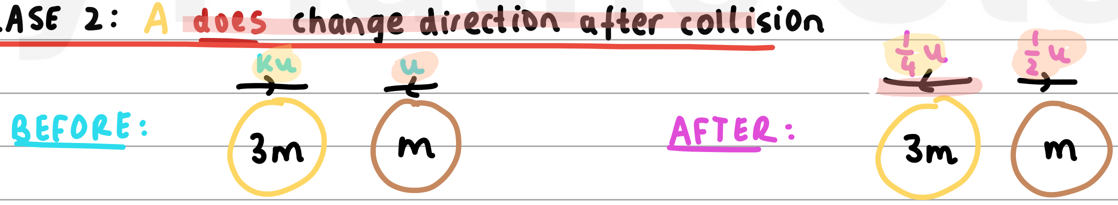
cancel u's and solve for 'k'

$$\Rightarrow 3k = \tfrac{9}{4}$$

$\div 3$ $\div 3$

$$\boxed{k = \tfrac{3}{4}}$$

CASE 2: A does change direction after collision



formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

Sub into above

$$3\cancel{m}(ku) + \cancel{m}(-u) = 3\cancel{m}\left(-\tfrac{1}{4}u\right) + \cancel{m}\left(\tfrac{1}{2}u\right)$$

cancel the m's and expand out:

$$3ku - u = -\tfrac{3}{4}u + \tfrac{1}{2}u$$

Question 1 continued

collect like terms:

$$3k\cancel{u} = \tfrac{3}{4}\cancel{u}$$

cancel u's and solve for 'k'

÷3 ÷3

$$\boxed{\Rightarrow k = \tfrac{1}{4}}$$

∴ two possible values of 'k':

$$\boxed{k = \tfrac{1}{4}, \tfrac{3}{4}}$$

Question 1 continued

(Total for Question 1 is 8 marks)

2.

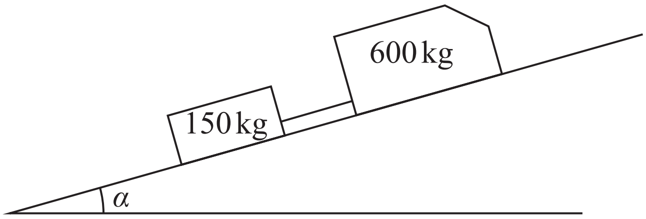


Figure 1

A van of mass 600 kg is moving up a straight road which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{15}$. The van is towing a trailer of mass 150 kg. The van is attached to the trailer by a towbar which is parallel to the direction of motion of the van and the trailer, as shown in Figure 1.

The resistance to the motion of the van from non-gravitational forces is modelled as a constant force of magnitude 200 N.
The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 100 N.

The towbar is modelled as a light rod.

The engine of the van is working at a constant rate of 12 kW.

Find the tension in the towbar at the instant when the speed of the van is 9 m s^{-1}

(8)

let's illustrate the above information on a detailed force diagram - label the respective weights, resistance to motion, the tension and the power rearranged

formula: $P = Fv$ (POWER in Watts, FORCE in Newtons, VELOCITY in ms^{-1})

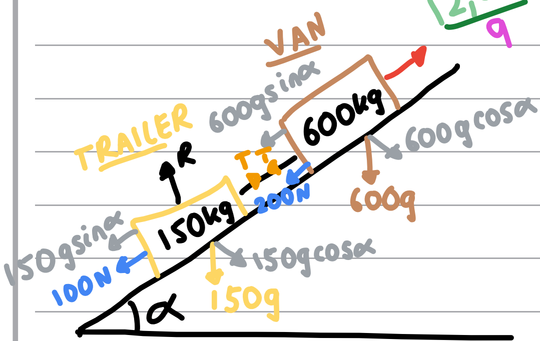
NOTE: could've calculated this power as a separate line of working but much more efficient in the exam to write in the FORCE from the POWER rearranged

$\Rightarrow F = \frac{P}{v}$

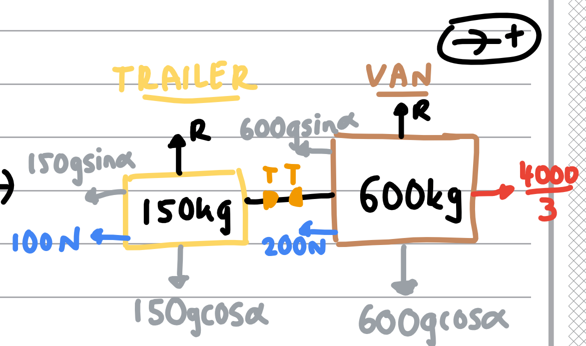
... where: $P = 12\text{kW} \xrightarrow{\text{convert to Watts} \times 1000} 12{,}000\text{W}$

and $v = 9$

$\frac{12{,}000}{9} = \frac{4000}{3}$



turn the axis



Question 2 continued

Remembering from Yr 2 Mechanics Chp 8 that if consider the system as a whole, the tension in the rod will cancel out - even though this isn't immediately helpful to us to get us that tension, it will help us get the acceleration we need to finally get the tension in the towbar

...hence using Newton's Second Law onto the system to get the acceleration:

formula: $\Sigma F_x = ma$

$R(\rightarrow): \ \frac{4000}{3} - 200 - 100 - 150g\sin\alpha - 600g\sin\alpha = (150+600)a$

collect like terms

$$\frac{3100}{3} - 750g\sin\alpha = 750a$$

subbing $\sin\alpha = 1/15$ from the question

$$\frac{3{,}100}{3} - 750g(1/15) = 750a$$

expand and solve for 'a':

$$\frac{3{,}100}{3} - 50g = 750a$$

$$750a = \frac{1630}{3}$$

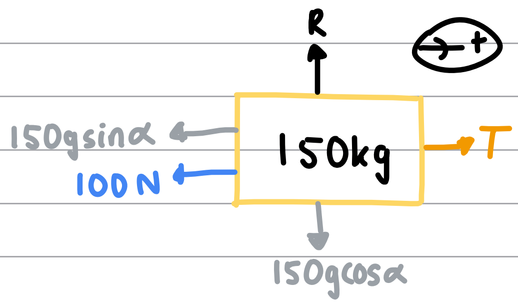
$\div 750 \qquad \div 750$

$$a = \frac{163}{225}$$

NOTE: keep this as an exact fraction to avoid rounding errors later on

now that we know the value for 'a', we can use Newton's Second Law - but this time on any of the trailer or the van - and solve for T

eg. on the trailer (less working as no power):



formula: $\Sigma F_x = ma$

$R(\rightarrow): \ T - 150g(1/15) - 100 = 150\left(\frac{163}{225}\right)$

expand

$$T - 10g - 100 = 326/3$$

$$\Rightarrow \ T = \frac{326}{3} + 10(9.8) + 100$$

Question 2 continued

$\therefore T = \frac{920}{3} = 306.66..$

$\boxed{= 307\text{ N}}$

(to 3 s.f)

Question 2 continued

(Total for Question 2 is 8 marks)

3.

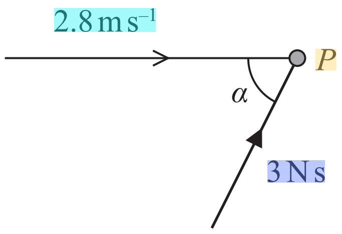


Figure 2

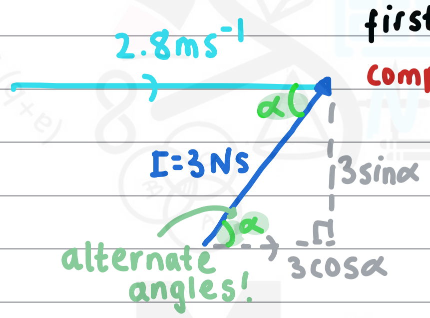
A particle P of mass 0.5 kg is moving in a straight line with speed 2.8 m s^{-1} when it receives an impulse of magnitude 3 N s.
The angle between the direction of motion of P immediately before receiving the impulse and the line of action of the impulse is α, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 2.

Find the speed of P immediately after receiving the impulse.

(5)

The fact that we're dealing with impulse at angles implies that we need to use the vector version of the Impulse-momentum principle

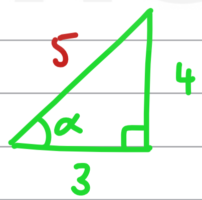
METHOD 1: subbing into the formula



first, resolving our Impulse of magnitude 3Ns into $\underline{i}$-$\underline{j}$ components ready to sub into our formula

$$\Rightarrow \; I = \begin{pmatrix} 3\cos\alpha \\ 3\sin\alpha \end{pmatrix}$$

but given in question that $\tan\alpha = 4/3$, hence exploiting the 3-4-5 Pythag. triple to form an appropriate trig triangle



$$\Rightarrow \sin\alpha = \frac{O}{H} = 4/5$$

$$\Rightarrow \cos\alpha = \frac{A}{H} = 3/5$$

$$\Rightarrow \; I = \begin{pmatrix} 3\,(3/5) \\ 3\,(4/5) \end{pmatrix}$$

$$= \begin{pmatrix} 9/5 \\ 12/5 \end{pmatrix} \text{Ns}$$

subbing the impulse, and the initial velocity as $\begin{pmatrix} 2.8 \\ 0 \end{pmatrix}$ms^{-1}, into our impulse-momentum formula:

(let $\underline{v} = \begin{pmatrix} a \\ b \end{pmatrix}$)

P 7 2 0 9 2 R A 0 1 0 3 2

formula: $I = m(\underline{v} - \underline{u})$

$$\begin{pmatrix} 9/5 \\ 12/5 \end{pmatrix} = 0.5\left(\begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} 2.8 \\ 0 \end{pmatrix}\right)$$

$\times 2$ $\times 2$

$$\begin{pmatrix} 18/5 \\ 24/5 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} - \begin{pmatrix} 2.8 \\ 0 \end{pmatrix}$$

rearrange for $\begin{pmatrix} a \\ b \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 18/5 + 2.8 \\ 24/5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 32/5 \\ 24/5 \end{pmatrix}$$

$$\therefore \underline{v} = \begin{pmatrix} 32/5 \\ 24/5 \end{pmatrix}$$

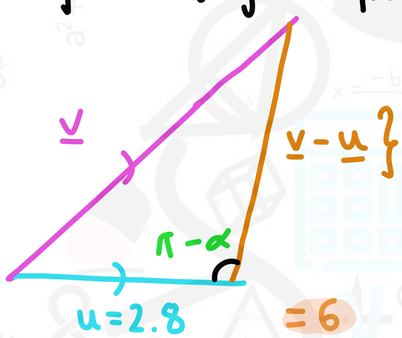
and using Pythagoras' to get the speed

$$v = \sqrt{(32/5)^2 + (24/5)^2}$$

$$= \sqrt{64} = \boxed{8\,ms^{-1}}$$

METHOD 2: forming a vector triangle

turning the diagram from Figure 2 into a vector triangle



get value of this from the formula for Impulse

formula: $I = m(\underline{v} - \underline{u})$

$3 = \frac{1}{2}(\underline{v} - \underline{u})$

$\Rightarrow \underline{v} - \underline{u} = 6$

and find the value for v using cosine rule:

formula: $c^2 = a^2 + b^2 - 2bc\cos A$

Subbing into above:

$$v^2 = (6)^2 + (2.8)^2 - 2(6)(2.8)\cos(\pi - \alpha)$$

expand fully

$$v^2 = 36 + 7.84 - 33.6\cos(\pi - \alpha)$$

↳ use cos addition formula

formula: $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$v^2 = 36 + 7.84 - 33.6(\cos\pi\cos\alpha + \sin\pi\sin\alpha)$$

collect like terms

$$v^2 = 36 + 7.84 - 33.6(-1(3/5) + 0)$$

$$\Rightarrow v^2 = 36 + 7.84 + 20.16$$

$$\Rightarrow v^2 = 64$$

square root

Question 3 continued

$\boxed{v = 8\,ms^{-1}}$

(Total for Question 3 is 5 marks)

4.

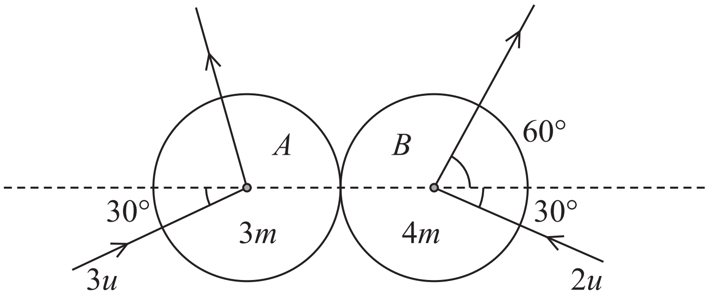


Figure 3

Two smooth uniform spheres, A and B, have equal radii. The mass of A is $3m$ and the mass of B is $4m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before they collide, A is moving with speed $3u$ at 30° to the line of centres of the spheres and B is moving with speed $2u$ at 30° to the line of centres of the spheres. The direction of motion of B is turned through an angle of 90° by the collision, as shown in Figure 3.

(i) Find the size of the angle through which the direction of motion of A is turned as a result of the collision.

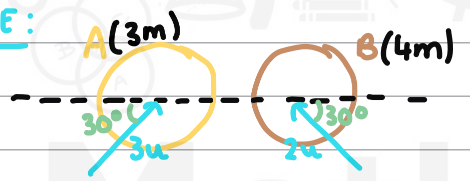
(ii) Find, in terms of m and u, the magnitude of the impulse received by B in the collision.

(9)

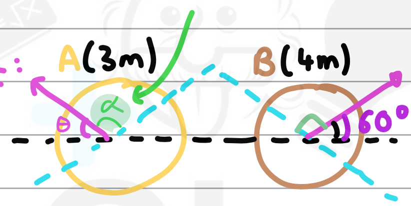
notice we have an 'oblique collisions between two spheres' question - splitting the diagram in Figure 3 into 'before' and 'after'

angle of deflection

BEFORE:



AFTER:



resolving into i-j components

resolving into i-j components

$3u\cos30° = \frac{3\sqrt{3}}{2}u$ $\quad 2u\cos30° = 2u\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}u$

BEFORE:

A $\quad$ B

$3u\sin30° = \frac{3}{2}u$ $\quad$ 3m $\quad$ 4m $\quad$ $2u\sin30° = u$

AFTER:

$x \quad y$

$\frac{3}{2}u$ $\quad$ 3m $\quad$ 4m $\quad$ u

no impulse along the line of centres ∴ perp. components remain THE SAME

because we want the angle of deflection for A, we need to find its final velocity

Question 4 continued

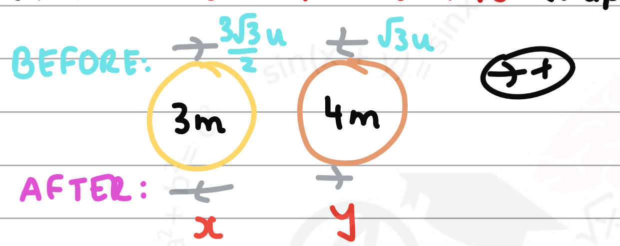
...perpendicular component: $\frac{3}{2}u$

...parallel component:

2 ways of finding this:

METHOD 1: labelling the parallel comp. as 'x' and 'y'

know that the impulse from the oblique collision acts parallel to line of centres $\therefore$ velocity does change - becomes a standard 'elastic collisions in 1D' set up



using PCLM to get 'x' - i.e total momentum before equals total momentum after

formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$$3\cancel{m}\left(\frac{3\sqrt{3}}{2}u\right) + 4\cancel{m}(-\sqrt{3}u) = 3\cancel{m}(-x) + 4\cancel{m}(y)$$

cancel m's and expand brackets

$$-3x + 4y = \frac{9\sqrt{3}}{2}u - 4\sqrt{3}u$$

$$\Rightarrow -3x + 4y = \frac{\sqrt{3}}{2}u \quad ①$$

next, using fact that B is turned through angle 90°

$$\Rightarrow \tan 60° = \frac{u}{y}$$

from the given 'AFTER' diagram

$$\Rightarrow y = \frac{u}{\tan 60°}$$

$$\therefore y = \frac{u}{\sqrt{3}}$$

$$\Rightarrow y = \frac{\sqrt{3}}{3}u$$

subbing into ① to get 'x'

$$-3x + 4\left(\frac{\sqrt{3}}{3}u\right) = \frac{\sqrt{3}}{2}u$$

expand brackets

METHOD 2: labelling the parallel comps relative to the angles 'θ' and '60°'

know that the impulse from the oblique collision acts parallel to the line of centres $\therefore$ velocity does change - becomes typical elastic collision in 1D question

using PCLM i.e the total momentum before equals total momentum after

formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$$3\cancel{m}\left(\frac{3\sqrt{3}}{2}u\right) + 4\cancel{m}(-\sqrt{3}u) = 3\cancel{m}(-x\cos\theta) + 4\cancel{m}(y\cos 60°)$$

cancel m's and evaluate $\cos 60° = \frac{1}{2}$

$$-3x\cos\theta + 2y = \frac{9\sqrt{3}}{2}u - 4\sqrt{3}u$$

collect like u's

$$-3x\cos\theta + 2y = \frac{\sqrt{3}}{2}u \quad ①$$

... perpendicular comps: no impact, so:

...for A:

$$\frac{3}{2}u = x\sin\theta$$

$\div \sin\theta$ $\quad \div \sin\theta$

$$\Rightarrow x = \frac{3u}{2\sin\theta} \quad ②$$

...for B:

$$u = y\sin 60°$$

$$\Rightarrow y = \frac{2\sqrt{3}}{3}u \quad ③$$

sub ② and ③ into ①

Question 4 continued

$$-3x + \frac{4\sqrt{3}}{3}u = \frac{\sqrt{3}}{2}u$$

collect like terms

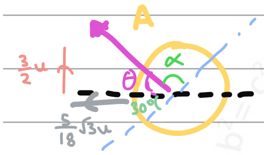
$$-3x = -\tfrac{5}{6}\sqrt{3}u$$

$\div -3$ $\quad \div -3$

$$x = \frac{5}{18}\sqrt{3}u$$

$$\therefore v_A = \begin{pmatrix} -\frac{5}{18}\sqrt{3}u \\ \frac{3}{2}u \end{pmatrix}$$

↳ populating this onto our initial 'AFTER' diagram for A



$$\therefore \tan\theta = \frac{\frac{3}{2}\cancel{u}}{\frac{5}{18}\sqrt{3}\cancel{u}}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{9\sqrt{3}}{5}\right)$$

$$\Rightarrow \theta = 72.21634..$$

$$\frac{\sqrt{3}}{2}u = -3\left(\frac{3u}{2\sin\theta}\right)\cos\theta + 2\left(\frac{2\sqrt{3}}{3}u\right)$$

expand brackets

$$\frac{\sqrt{3}}{2}\cancel{u} = -\frac{9\cancel{u}\cos\theta}{2\sin\theta} + \frac{4\sqrt{3}}{3}\cancel{u}$$

cancel u's and use $\frac{\cos\theta}{\sin\theta} \equiv \cot\theta$

$$\frac{\sqrt{3}}{2} = -\frac{9}{2}\cot\theta + \frac{4\sqrt{3}}{3}$$

Solve for $\cot\theta$:

$$\cot\theta = \frac{5\sqrt{3}}{27}$$

rationalise

$$\tan\theta = \frac{1}{\frac{5\sqrt{3}}{27}} = \frac{27}{5\sqrt{3}} \frac{\times\sqrt{3}}{\times\sqrt{3}}$$

$$= \frac{\cancel{27}^{9}\sqrt{3}}{\cancel{15}_{5}}$$

$$\therefore \tan\theta = \frac{9\sqrt{3}}{5}$$

$$\therefore \theta = \tan^{-1}\left(\frac{9\sqrt{3}}{5}\right)$$

$$= 72.2163..$$

but we're not asked for the angle θ - need the angle α - which, looking at our initial 'AFTER' diagram is

$$\alpha = 180° - 72.216...° - 30°$$

$$= 77.7836...$$

$$\boxed{= 78° \text{ (to 2.s.f)}}$$

(ii) remembering that because for oblique collisions, the Impulse only acts parallel to the line of centres, we only need to sub in the parallel comps of B into the Impulse-momentum formula

BEFORE $\quad \xleftarrow{\sqrt{3}u}$

(4m) $\quad$ ($\rightarrow +$)

AFTER $\quad \rightarrow$

$\frac{\sqrt{3}}{3}u$ or if used METHOD 2: $y\cos 60°$

METHOD 1

$$\frac{2\sqrt{3}}{3}u\left(\frac{1}{2}\right) = \sqrt{3}/_3\, u$$

formula: $I = m(v-u)$

$$I = 4m\left(\frac{\sqrt{3}}{3}u - (-\sqrt{3}u)\right)$$

$$I = 4m\left(\frac{4\sqrt{3}}{3}u\right)$$

$$\boxed{\therefore I = \frac{16\sqrt{3}}{3}mu \text{ Ns}}$$

P 7 2 0 9 2 R A 0 1 4 3 2

Question 4 continued

(Total for Question 4 is 9 marks)

5. Two particles, P and Q, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.
The mass of P is $3m$ and the mass of Q is $4m$.
Immediately before the collision the speed of P is $2u$ and the speed of Q is u.
The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{u}{7}(9e+2)$

(6)

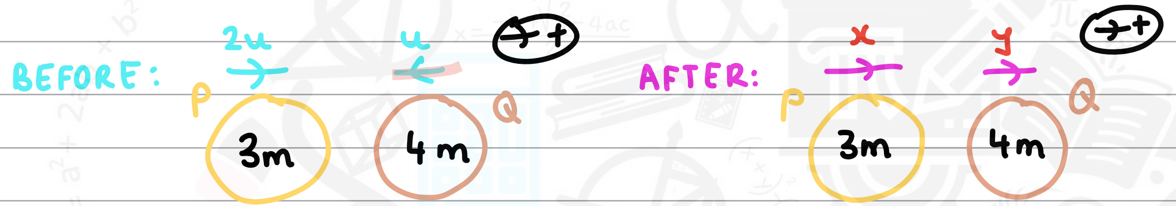
After the collision with P, particle Q collides directly with a fixed vertical wall and rebounds. The wall is perpendicular to the direction of motion of Q.

The coefficient of restitution between Q and the wall is $\frac{1}{2}$

(b) Find the complete range of possible values of e for which there is a second collision between P and Q.

(4)

(a) first part of the question is a typical 'elastic collisions in 1D' question - illustrating it diagrammatically - label direction of motion, respective speeds etc.



following the normal procedure for these types of collision: notice how both the velocities after are unknown, so can't just stop at using PCLM - have to do NEL (Impact law) as well:

... first, PCLM - means total momentum before equals total momentum after:

formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

sub into above

$3\cancel{m}(2u) + 4\cancel{m}(-u) = 3\cancel{m}(x) + 4\cancel{m}(y)$

cancel m's and expand brackets

$3x + 4y = 6u - 4u$

$3x + 4y = 2u$ — ①

... next, NEL:

formula $e = \frac{\text{speed of separation}}{\text{speed of approach}}$

$e = \frac{v_B - v_A}{u_A - u_B}$

Question 5 continued

subbing into above:

$$e = \frac{y - x}{2u - (-u)}$$

$$\Rightarrow e = \frac{y-x}{3u}$$

$\times 3u$ $\quad\times 3u$

$$y - x = 3eu \quad - ②$$

need to solve ① and ② simultaneously - elim. 'x'

① + ② × 3

$$3x + 4y = 2u$$
$$^{+}\,\underline{-3x + 3y = 9eu}$$
$$\underline{7y = 2u + 9eu}$$

factorise 'u' out

$$7y = u(2 + 9e)$$

$\div 7$ $\quad\div 7$

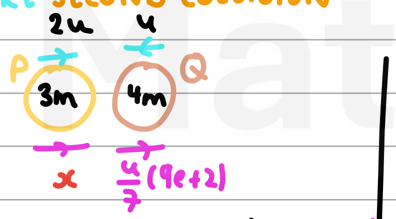
$$y = \frac{u}{7}(2 + 9e)$$

$$\boxed{\therefore v_Q = \frac{u}{7}(2+9e) \text{ as required}}$$

(b) now we're introduced to a second collision - this time the one between particle Q and a vertical wall

↳ illustrating this diagrammatically:

BEFORE SECOND COLLISION



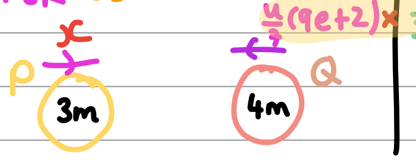
$\rightarrow +$

$e = 1/2$

...to get v_Q after second collision - multiply initial speed by e:

AFTER SECOND COLLISION

$\frac{u}{7}(9e+2) \times \frac{1}{2} = \frac{u}{14}(9e+2)$



can infer from the diagram that the only way there'll ever be a second collision between P and Q is if $-\frac{u}{14}(9e+2) < x$ - this is because we are moving in

Question 5 continued

the -ve direction, so we want the v_B to be even more -ve so we catch up with x

...finding 'x':

get the ① and ② from part (a):

$3x+4y=2u$ ①

$-x+y=3eu$ ②

and now need 'x', so elim. 'y'

① $-4\times$②

$$3x+4y=2u$$
$$-\ \ -4x+4y=12eu$$
$$7x=2u-12eu$$

factorise 'u'

$$7x=u(2-12e)$$

$\div 7$ $\quad\div 7$

$$x=\frac{u}{7}(2-12e)$$

↳ assume this is -ve and x is going left (collision logic)

∴ subbing into our source of inequality

$$-\frac{u}{14}(9e+2)<\frac{u}{7}(2-12e)$$

$\times -14$ $\quad \times -14$ (hence changes sign)

$$\cancel{u}(9e+2)>-2\cancel{u}(2-12e)$$

cancel 'u's'

$$9e+2>-4+24e$$

collect like terms

$$15e<6$$

$$\Rightarrow e<6/15 \xrightarrow{\div 3} e<2/5$$

but know $0\le e\le 1$, so full range is

$$\boxed{0<e<2/5}$$

Question 5 continued

(Total for Question 5 is 10 marks)

6.

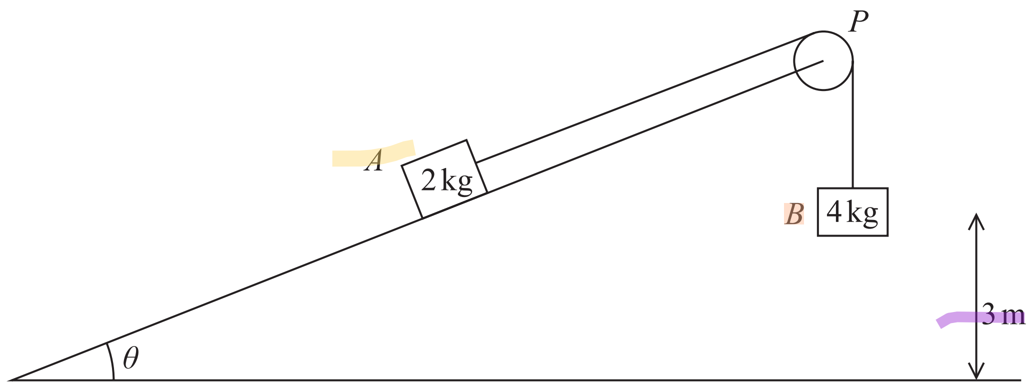


Figure 4

Two blocks, A and B, of masses 2 kg and 4 kg respectively are attached to the ends of a light inextensible string.

Initially A is held on a fixed rough plane. The plane is inclined to horizontal ground at an angle θ, where $\tan\theta = \frac{3}{4}$

The string passes over a small smooth light pulley P that is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane.

Block A is held on the plane with the distance AP greater than 3 m.
Block B hangs freely below P at a distance of 3 m above the ground, as shown in Figure 4.

The coefficient of friction between A and the plane is μ

Block A is released from rest with the string taut.

By modelling the blocks as particles,

(a) find the potential energy lost by the whole system as a result of B falling 3 m.

(3)

Given that the speed of B at the instant it hits the ground is $4.5\,\text{m s}^{-1}$ and ignoring air resistance,

(b) use the work-energy principle to find the value of μ

(6)

After B hits the ground, A continues to move up the plane but does not reach the pulley in the subsequent motion.
Block A comes to instantaneous rest after moving a total distance of $(3 + d)$ m from its point of release.

Ignoring air resistance,

(c) use the work-energy principle to find the value of d

(4)

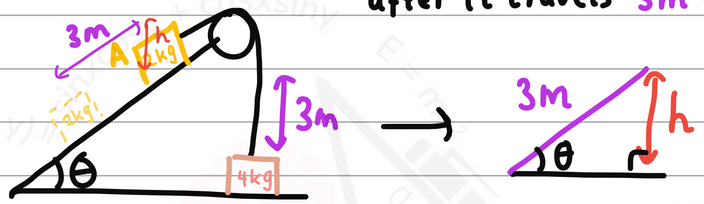
(a) it looks like the part (a) of this conservation of mechanical energy question only wants us to look at the G.P.E lost by the system as B falls 3m

Question 6 continued

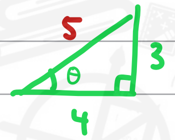
we can see from Figure 4 that:

$$\text{potential energy lost by system} = \underbrace{\text{G.P.E lost by B}}_{\text{vertical fall}} - \underbrace{\text{G.P.E gained by A}}_{\text{travels up the inclined plane}}$$

↳ for A, need the perp. height, h, after it travels 3m up the plane



we're given in the question that $\tan\theta = 3/4$ - hence exploiting the 3-4-5 Pythag. triple to get a trig triangle:



$\Rightarrow \sin\theta = \frac{O}{H} = 3/5$

$\Rightarrow \cos\theta = \frac{A}{H} = 4/5$

$\therefore$ know that $h = 3\sin\theta$
$= 3(3/5)$
$= 9/5$

now subbing this into our formula for G.P.E:

formula: $E_p = mgh$ (m: mass in kg; g: gravitational field strength in Nkg^{-1}; h: height in m)

potential energy lost by system $= 4(9.8)(3) - 2(9.8)(9/5)$
$= 117.6 - 35.28$
$= 82.32$ or $\boxed{82.3\text{ J (3 s.f)}}$

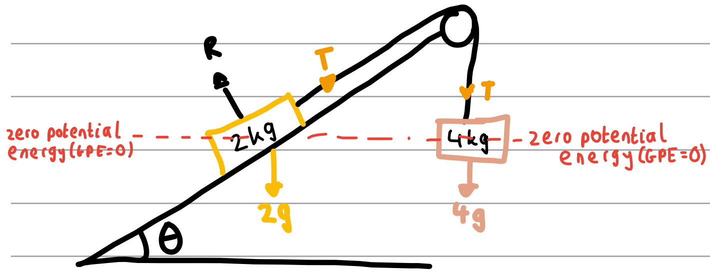
(b) let's illustrate the 'before' and 'after' of B falling 3m (hence A moving 3m up the plane) and label the appropriate energies

Question 6 continued

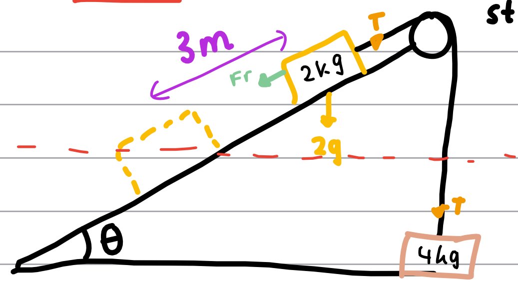
- $G.P.E$ lost (part (a))
- $K.E$ gained ($v = 4.5ms^{-1}$)

↳ both particles as connected by light inextensible string

BEFORE



AFTER

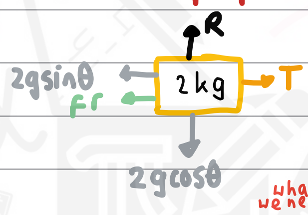


- work done by friction:

formula: $Fr \times d$ (Fr: friction; d: distance travelled)

formula: $Fr = \mu R$ (μ: coeff. of friction — what we need to find!; R: reaction force)

↳ can get by resolving A perp. to plane



$R(\perp)$:

$R = 2g\cos\theta$ (from trig triangle in (a))

$= 2g(4/5)$

$= 8g/5$

$\therefore Fr = \mu \, 8g/5$ (what we need to find!)

hence subbing into a more 'PHYSICS' version of the formula for work-energy principle – i.e one that allows us to utilise the G.P.E lost by system that we'd calculated in part (a)

formula: energy lost = energy gained

$G.P.E \text{ lost} - w.d \text{ by friction} = K.E \text{ gained}$

(mgh, $Fr \times d$, $\frac{1}{2}mv^2$)

$$82.3 - \frac{8g\mu}{5}(3) = \frac{1}{2}(2+4)(4.5)^2$$

expand out

$$82.3 - \frac{24\mu g}{5} = 60.75$$

rearrange and solve for μ:

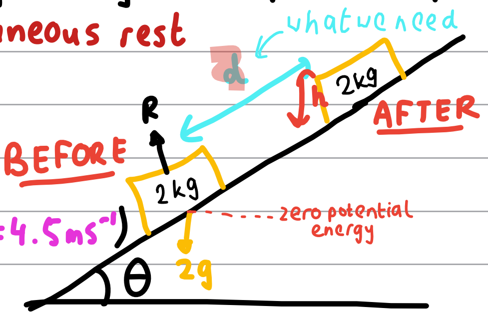
$$\frac{24\mu g}{5} = 21.55$$

$\div \frac{24}{5}$ $\quad$ $\div \frac{24}{5}$

$$\boxed{\begin{aligned}\mu &= 0.45812\ldots \\ &= 0.46 \text{ (to 2 d.p)}\end{aligned}}$$

Question 6 continued

(c) now focusing on the position of A after B falls and until B comes to instantaneous rest



• K.E ($v = 4.5\,ms^{-1}$)

• G.P.E
↳ need perp distance travelled by A

$\Rightarrow h = d\sin\theta$
$\Rightarrow h = d(3/5)$
$\therefore h = 3/5\, d$

• w.d by friction
formula: $Fr \times d$
formula: $Fr = \mu R$
saw from (b) that $Fr = \frac{8g}{5}\mu$ → now can sub in $\mu = 0.4581...$ $\therefore Fr = \frac{431}{60}$

now sub all into work-energy principle (includes dissipative forces)

w.d in (n/a) + K.E_i (initial kinetic) + G.P.E_i (initial gravitational potential) + E.P.E_i (initial elastic potential) = K.E_f (final kinetic energy) + G.P.E_f (final gravitational potential) + E.P.E_f (final elastic potential) + w.d against friction

formula: $\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l} + Fr \times d$

subbing into above:

$$\frac{1}{2}(2)(4.5)^2 + 0 + 0 = 0 + 2(9.8)(3/5\,d) + 0 + \frac{431}{60}d$$

expand

$$\frac{81}{4} = \frac{294}{25}d + \frac{431}{60}d$$

collect like terms

$$\frac{5683}{300}d = 81/4$$

$\div \frac{5683}{300}$ $\quad$ $\div \frac{5683}{300}$

$$d = 1.06897..$$

$$\boxed{= 1.07\,(3\,s.f)\,m}$$

(Total for Question 6 is 13 marks)

7. A spring of natural length a has one end attached to a fixed point A. The other end of the spring is attached to a package P of mass m.
The package P is held at rest at the point B, which is vertically below A such that $AB = 3a$.
After being released from rest at B, the package P first comes to instantaneous rest at A.
Air resistance is modelled as being negligible.

By modelling the spring as being light and modelling P as a particle,

(a) show that the modulus of elasticity of the spring is $2mg$

(5)

(b) (i) Show that P attains its maximum speed when the extension of the spring is $\frac{1}{2}a$

(ii) Use the principle of conservation of mechanical energy to find the maximum speed, giving your answer in terms of a and g.

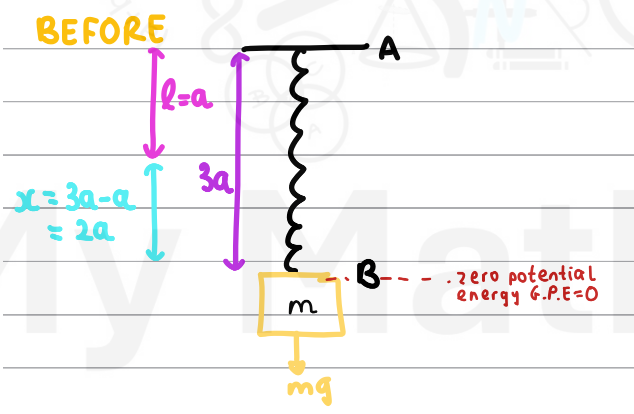
(6)

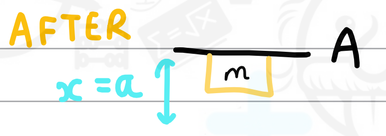
In reality, the spring is not light.

(c) State one way in which this would affect your energy equation in part (b).

(1)

as with every 'elastic strings and springs' question, the most important thing is to draw a detailed diagram - here, one for before the string is released, second for when it comes to instantaneous rest - and label with appropriate energies





· E.P.E – "stretched"

· E.P.E – "compressed"
· G.P.E

now sub all into work-energy principle (includes dissipative forces)

$$\text{w.d in} + K.E_i + G.P.E_i + E.P.E_i = K.E_f + G.P.E_f + E.P.E_f + \text{w.d against friction}$$

w.d in: n/a; $K.E_i$: initial kinetic; $G.P.E_i$: initial gravitational potential; $E.P.E_i$: initial elastic potential; $K.E_f$: final kinetic energy; $G.P.E_f$: final gravitational potential; $E.P.E_f$: final elastic potential

formula: $\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l} + Fr \times d$

Question 7 continued

Subbing into above : what we need

$$0 + 0 + \frac{\lambda(2a)^2}{2a} = 0 + mg(3a) + \frac{\lambda(a)^2}{2a} + 0$$

expand out

$$2\cancel{a}\lambda = 3mg\cancel{a} + \frac{\lambda \cancel{a}}{2}$$

cancel 'a's and rearrange to solve for λ

$$\frac{3}{2}\lambda = 3mg$$

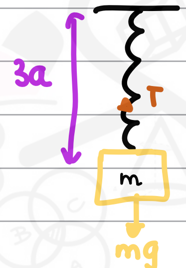
$\div 3/2$ $\quad$ $\div 3/2$

$\boxed{\lambda = 2mg}$ as required

(b)(i) METHOD 1: using equilibrium

here need to interpret the point where the particle reaches max speed as the point where the spring is in equilibrium i.e forces up = forces down

... looking at the force diagram for the spring:



$R(\uparrow)$: $T = mg$

hence, need to find the extension at which the above equilibrium condition applies (and see if it's $x = \frac{1}{2}a$)

... subbing into the formula for tension in strings/springs:

formula: $T = \frac{\lambda x}{l}$ (λ mod. of elasticity, x ← extension in m, l ← natural length in 'm')

$$\cancel{m}\cancel{g} = \frac{2\cancel{m}\cancel{g}(x)}{a}$$

$\times a$ $\quad$ $\times a$

cancel m's and g's

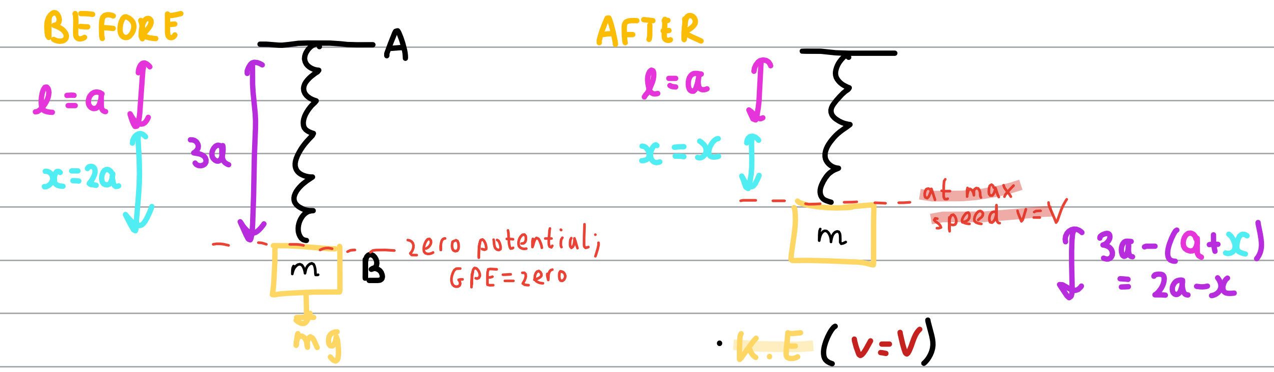
$$a = 2x$$

$\boxed{\Rightarrow x = \frac{a}{2} \text{ as required}}$

METHOD 2 : using principle of conservation of mechanical energy and differentiation

↳ key here is to draw two further diagrams – first when P is held at rest at the point B, second when particle is at $x = x$ (this will be the extension for which package is at max speed, V) and label appropriate energies

Question 7 continued



· E.P.E

· K.E (v=V)

· E.P.E

· G.P.E

now sub all into work-energy principle (includes dissipative forces)

$$\text{w.d in} + K.E_i + G.P.E_i + E.P.E_i = K.E_f + G.P.E_f + E.P.E_f + \text{w.d against friction}$$

w.d in: n/a; $K.E_i$: initial kinetic; $G.P.E_i$: initial gravitational potential; $E.P.E_i$: initial elastic potential; $K.E_f$: final kinetic energy; $G.P.E_f$: final gravitational potential; $E.P.E_f$: final elastic potential

formula: $$\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l} + Fr\times d$$

subbing into above: from (a)

$$0 + 0 + 2a(2\cancel{m}g) = \frac{1}{2}\cancel{m}V^2 + \cancel{m}g(2a-x) + \frac{(2\cancel{m}g)(x)^2}{2a}$$

cancel m's and expand

$$4ag = \frac{1}{2}V^2 + 2ag - xg + \frac{gx^2}{a}$$

collect like terms and rearrange to solve for V^2

$$\frac{1}{2}V^2 = 2ag - \frac{gx^2}{a} + xg$$

$\times 2$ $\qquad$ $\times 2$

$$V^2 = 4ag - \frac{2gx^2}{a} + 2xg$$

now exploit fact that max speed means $\frac{dV^2}{dx} = 0$

hence differentiating above:

$$\frac{dV^2}{dx} = -\frac{4\cancel{g}x}{a} + 2\cancel{g} = 0$$

cancel g's and solve for x

$$\frac{4x}{a} = 2$$

$\div 4/a$ $\qquad$ $\div 4/a$

$$\Rightarrow \boxed{x = \frac{2a}{4} = \frac{a}{2} \text{ as required}}$$

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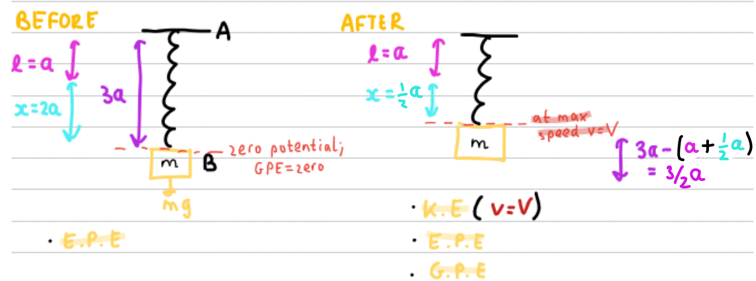
(ii) now the benefit of using Method 2 for (i) is that now for (ii) you'd just sub $x = \frac{1}{2}a$ into the final simplified equtn (the one you'd differentiated) and solve for V

-however, if used Method 1, now you'd use the work energy principle with $x = \frac{1}{2}a$



now sub all into work-energy principle (includes dissipative forces)

w.d in + $K.E_i$ + $G.P.E_i$ + $E.P.E_i$ = $K.E_f$ + $G.P.E_f$ + $E.P.E_f$ + w.d against friction

w.d in → n/a; $K.E_i$ → initial kinetic; $G.P.E_i$ → initial gravitational potential; $E.P.E_i$ → initial elastic potential; $K.E_f$ → final kinetic energy; $G.P.E_f$ → final gravitational potential; $E.P.E_f$ → final elastic potential

formula: $\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l} + F_r \times d$

Sub into above :

$$0 + 0 + \frac{2mg(2a)^2}{2(a)} = \frac{1}{2}mV^2 + mg\left(\frac{3}{2}a\right) + \frac{2mg\left(\frac{a}{2}\right)^2}{2a} + 0$$

expand

$$4ag\cancel{m} = \frac{1}{2}\cancel{m}V^2 + \frac{3}{2}ag\cancel{m} + \frac{1}{4}ag\cancel{m}$$

cancel m's and collect like terms

$$\frac{1}{2}V^2 = \frac{9}{4}ag$$

×2 ×2

$$V^2 = \frac{9}{2}ag$$

squr root

$$\Rightarrow \boxed{V = 3\sqrt{\frac{ag}{2}}}$$

(c) assumptions - particularly from Yr 1 Mechanics Chp 8 :

- would need to include G.P.E in energy because of the mass
- extension in the spring will be different

Question 7 continued

(Total for Question 7 is 12 marks)

8.

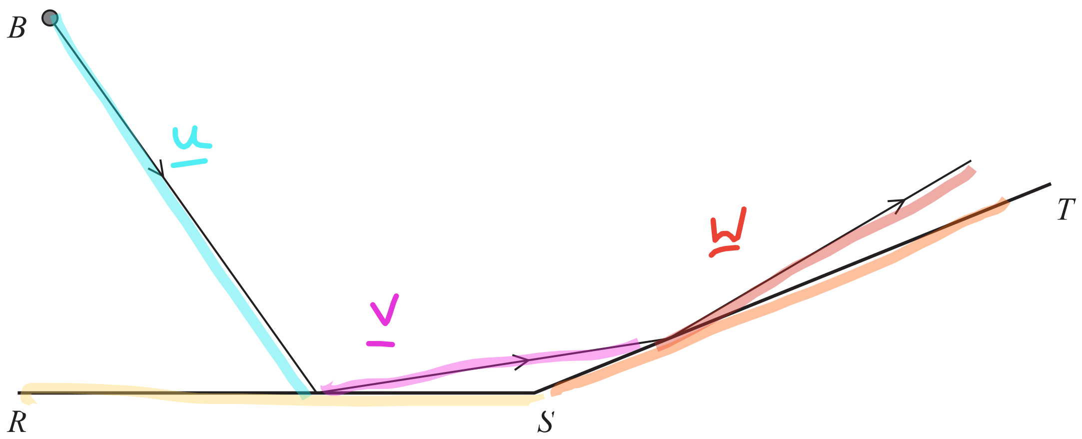


Figure 5

Figure 5 represents the plan view of part of a smooth horizontal floor, where RS and ST are smooth fixed vertical walls. The vector $\overrightarrow{RS}$ is in the direction of $\mathbf{i}$ and the vector $\overrightarrow{ST}$ is in the direction of $(2\mathbf{i} + \mathbf{j})$.

A small ball B is projected across the floor towards RS. Immediately before the impact with RS, the velocity of B is $(6\mathbf{i} - 8\mathbf{j})\,\mathrm{m\,s^{-1}}$. The ball bounces off RS and then hits ST.

The ball is modelled as a particle.

Given that the coefficient of restitution between B and RS is e,

(a) find the full range of possible values of e.

(3)

It is now given that $e = \frac{1}{4}$ and that the coefficient of restitution between B and ST is $\frac{1}{2}$

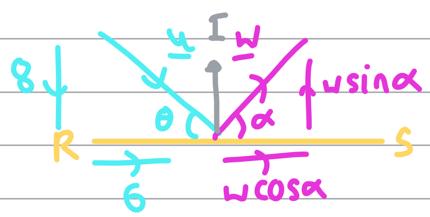
(b) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the velocity of B immediately after its impact with ST.

(7)

see from Figure 5 we have two successive oblique collisions with two vector walls

(a) let's focus on the FIRST COLLISION - one between the small ball and the wall RS - illustrating diagrammatically

FIRST COLLISION



... parallel components :

impulse doesn't act along the fixed surface, so no change in the velocities

$\Rightarrow w\cos\alpha = 6$

DO NOT WRITE IN THIS AREA

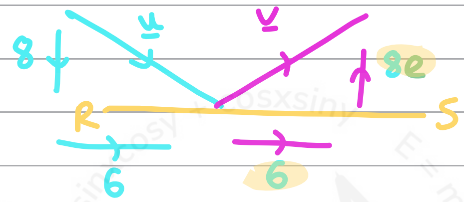
Question 8 continued

... perpendicular components:

impulse does act perp. to the fixed surface, hence the velocities are impacted by NEL rearranged

$\Rightarrow u\sin\alpha = 8e$

hence adding these onto the diagram:



$\therefore \underline{v} = \begin{pmatrix} 6 \\ 8e \end{pmatrix}$

now using our 'collisions logic' as our only source of inequality - for the ball to have a SECOND COLLISION - now with ST - the $\underline{v}$ vector must have a smaller $\underline{j}:\underline{i}$ ratio ('gradient') than the wall vector ST

$\Rightarrow \frac{8e}{6} < \frac{1}{2}$

$\times 6 \quad \times 6$

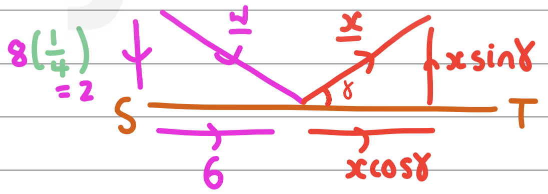
$8e < 3$

$\div 8 \quad \div 8$

$e < 3/8$

but know $0 \le e \le 1$, so full range is $\boxed{0 < e < 3/8}$

(b) sub fact that $e = 1/4$ into $\underline{v}$ as we illustrate the SECOND COLLISION diagrammatically - the one between the ball and wall ST



now to find this velocity $\underline{x} = \begin{pmatrix} a \\ b \end{pmatrix}$, use two formulae

formula: $u \cdot w = v \cdot w$ (vector wall)

$\begin{pmatrix} 6 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

evaluate scalar product

$12 + 2 = 2a + b$

$\Rightarrow 2a + b = 14$ — ①

Question 8 continued

impulse

formula : $-e\underline{u}\cdot\underline{I} = \underline{v}\cdot\underline{I}$

coeff. of restitution between ball and wall

but the impulse isn't given - just know it's perpendicular to our vector wall $\overrightarrow{RS} = \begin{pmatrix}2\\1\end{pmatrix}$

... 2 ways to do this:

WAY 1: dot product and by inspection

formula: $\underline{a}\cdot\underline{b} = 0$

$\begin{pmatrix}2\\1\end{pmatrix}\cdot\begin{pmatrix}a\\b\end{pmatrix} = 0$

$\therefore \underline{I} = \begin{pmatrix}-1\\2\end{pmatrix}$ or $\begin{pmatrix}1\\-2\end{pmatrix}$

WAY 2: gradients

$\underline{w} = \begin{pmatrix}2\\1\end{pmatrix} \rightarrow$ interpret this as the gradient of line $y = \frac{1}{2}x$

$\therefore m = \frac{1}{2}$

using $m_1 \times m_2 = -1$

$\therefore$ line $\perp$-ar is $y = -2x$

$\hookrightarrow$ vector $\begin{pmatrix}1\\-2\end{pmatrix}$

$\therefore \underline{I} = \begin{pmatrix}1\\-2\end{pmatrix}$

can use either - eg. $\underline{I} = \begin{pmatrix}1\\-2\end{pmatrix}$ - sub into formula:

$-\frac{1}{2}\begin{pmatrix}6\\2\end{pmatrix}\cdot\begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}\cdot\begin{pmatrix}1\\-2\end{pmatrix}$

evaluate scalar product

$-\frac{1}{2}(2) = a - 2b$

$\Rightarrow a - 2b = -1$ ②

now solve ① and ② simultaneously:

① $-2\times$②

$$\begin{aligned} 2a + b &= 14 \\ -\,2a - 4b &= -2 \\ \hline 5b &= 16 \end{aligned}$$

$\div 5 \qquad \div 5$

$\Rightarrow b = 16/5$ or 3.2

sub into ① for 'a'

$2a + (3.2) = 14$

$2a = 10.8$

$\div 2 \qquad \div 2$

$a = 5.4$

$\boxed{\therefore \underline{x} = \begin{pmatrix}5.4\\3.2\end{pmatrix} \text{ms}^{-1}}$

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS